

# COMP 350 — Theory of Computation

## Problem Set 02

September 19, 2017

Your examination of chapter 2 will revolve around the following problems. You may or may not do these problems exactly as stated in the text so read this document carefully. **Due Dates are given below. Completed assignments are due at 1pm on the listed dates. Failure to submit work or demonstrate significant progress towards the work by the given deadline will result in a zero for that part of the assignment. No exceptions will be made.**

### Exercises & Problems

2.2\*

2.7†

2.9

2.10

2.17\*

2.21\*

2.23†

2.24†

2.25†

2.27†

2.33\*

The work done on your language studies described below<sup>1</sup> will count towards the problem set part of your grade. The work described in the subsequent sections count towards the proof part of your grade.

<sup>1</sup> Examples, Classification, and NP-Completeness

### *Language Studies*

Exercises marked with an asterisk are about specific languages. Your work will start by examining these languages and their classifications in detail.

### *Examples of Key Languages*

**Due 9/19**

For each of the languages you should generate *three or more* instances of the language and another couple of examples from the language's complement. For example, if the language is 3SAT, then you'd come up with three or more 3-CNF instances that are satisfiable and two more more that are not satisfiable.

*Language Classification***Due 9/21**

Using the textbook and other sources as needed, determine which languages are in P, NP, EXP, or NEXP. For each language find or design an algorithm that verifies a candidate in exponential time. For any language that's in EXP, find or design an algorithm that determines language membership in exponential time. For any language that's in NP, find or design a polynomial time verification algorithm. Finally, for any language that is in P, find or design an algorithm that determines language membership in polynomial time.

Algorithms can be specified in high-level psuedo-code and need not be worked out down to the Turing-Machine level. You should be prepared to demonstrate the algorithms using the examples of the languages and their compliments that you've generated.

*NP-Completeness***Due 9/26**

Using the textbook and other sources, determine which of the languages is NP-Complete. For those languages, determine if there are known Karp reductions between them and draw a web of reductions specific to this language set<sup>2</sup>. If no immediate reductions appear to exist, find a shortest path<sup>3</sup> through other languages such that you have a connected web of reductions. Finally, find out if there are known Karp reductions from those languages to SAT and include SAT in your web.

Choose one or two reductions<sup>4</sup> apply them to some of your examples.

<sup>2</sup> See Figure 2.4<sup>3</sup> not provably shortest, but just short<sup>4</sup> or all of them*Class Relationships and Definitions***Due 9/28**

Problems marked with a dagger get at relationships between complexity classes and consequences of P vs NP. These should be completed as directed in the text but should also be contextualized and demonstrated with the languages you studied and the examples of those languages. If no such demonstration can be found with your languages set, then try to determine what additional languages might be studied in order to demonstrate the issue at hand<sup>5</sup>.

<sup>5</sup> You don't actually need to study the new languages but certainly can if you want.*Properties of Language Operations***Due 9/28**

The remaining problems, 9 and 10, are about properties of reductions, set union, and set intersection. Complete these problems as directed but also contextualize and demonstrate using the languages you studied and the examples of those languages you generated. If no such demonstration can be found with your languages set, then try to determine what additional languages might be studied in order to demonstrate the issue at hand<sup>6</sup>.

### *Project Preparation*

**Due 9/28<sup>7</sup>.**

In addition to working the problems, you'll be asked to carry out a more detailed examination and demonstration of Cook's Proof that SAT is NP-Complete<sup>8</sup>. Towards this end you should find and print Cook's original paper, seek out other textbook or tutorial presentations of Cook's proof, and seek out concrete demonstrations of Cook's reduction. Once we have these resources, we can determine if there's some other novel demonstration or tutorial we might pursue.

<sup>6</sup> You don't actually need to study the new languages but certainly can if you want.

<sup>7</sup> This work counts as the proof part of your grade for this chapter. All other work goes towards your problem set grade

<sup>8</sup> Section 2.3 of the text